

Weakly Supervised Learning of Deep Structured Models

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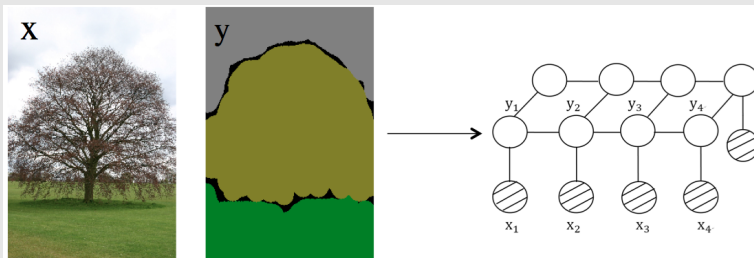
Outline

- 1 Context
- 2 Contributions
- 3 Results

Structured prediction

Structured inputs and outputs

- \mathcal{X} is the input space : arbitrary (non vectorial, etc)
- \mathcal{Y} is the structured output space: discrete, with variables strongly correlated \Rightarrow probabilistic graphical models (chain, tree, general graph)
- Ex: semantic image segmentation \Rightarrow classify each pixel into semantic categories
 - Output space $\mathcal{Y} = \{1, \dots, k\}^D$ with correlated variables



Structured prediction

Structural SVM (SSVM) [TJHA05]

- Relationship between input $\mathbf{x} \in \mathcal{X}$ and output $\mathbf{y} \in \mathcal{Y}$
 \Rightarrow joint feature map $\Psi(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^d$
- Scoring function linear in Ψ : $f_{\mathbf{w}}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{w}, \Psi(\mathbf{x}, \mathbf{y}) \rangle = s(\mathbf{y})$
 - Kernel extension possible
 - $\Psi(\mathbf{x}, \mathbf{y})$ possibly deep, WELDON or [CSYU15]
- Prediction or **inference**:
$$\hat{\mathbf{y}}(\mathbf{x}, \mathbf{w}) = \arg \max_{\mathbf{y} \in \mathcal{Y}} \langle \mathbf{w}, \Psi(\mathbf{x}, \mathbf{y}) \rangle = \arg \max_{\mathbf{y} \in \mathcal{Y}} s(\mathbf{y})$$
- Output space \mathcal{Y} generally huge \Rightarrow exhaustive maximization not tractable
 - Exploit structure (exact solutions for chain, trees), specific scoring functions (sub-modular), *etc*
 - Inference in graphical models: extremely rich literature

Structured prediction

Structural SVM: training

- Training: a set of N labeled trained pairs $(\mathbf{x}_i, \mathbf{y}_i)$
- Structured loss $\Delta(\hat{\mathbf{y}}_i, \mathbf{y}_i), \hat{\mathbf{y}}_i(\mathbf{x}_i, \mathbf{w}) = \arg \max_{\mathbf{y} \in \mathcal{Y}} \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}) \rangle$
 \Rightarrow *Prior* (expert) knowledge on the dissimilarity between two outputs
- Dependence of Δ wrt \mathbf{w} complex (non-convex, non-smooth)
- **Margin rescaling**: convex upper bound $\Delta(\hat{\mathbf{y}}_i, \mathbf{y}_i) \leq \ell(\mathbf{x}_i, \mathbf{y}_i, \mathbf{w})$
 $\ell(\mathbf{x}_i, \mathbf{y}_i, \mathbf{w}) = \max_{\mathbf{y} \in \mathcal{Y}} [\Delta(\mathbf{y}_i, \mathbf{y}) + \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}) \rangle] - \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}_i) \rangle$
- $\max_{\mathbf{y} \in \mathcal{Y}} [\Delta(\mathbf{y}_i, \mathbf{y}) + s(\mathbf{y})]$ "Loss Augmented Inference" (LAI) \Rightarrow
 exhaustive maximization not tractable
 - Generally harder than inference (depends on Δ)

Structured prediction

Structured Output Ranking

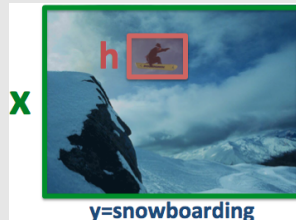
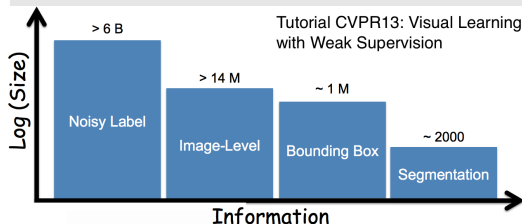
- Input \mathcal{X} list of n examples: $\mathbf{x} = (o_1, \dots, o_n)$
- Output \mathcal{Y} ranking of examples ($|\mathcal{Y}| \sim 2^{n^2/2}$): \mathbf{y} matrix s.t.

$$y_{ij} = \begin{cases} +1 & \text{if } o_i <_y o_j \text{ (} o_i \text{ is before } o_j \text{ in the sorted list)} \\ -1 & \text{if } o_i >_y o_j \text{ (} o_i \text{ is after } o_j \end{cases}$$
- Ranking feature map: $\Psi(\mathbf{x}, \mathbf{y}) = \sum_{i \in \Theta} \sum_{j \in \Theta} y_{ij} (\phi(o_i) - \phi(o_j))$
- **Inference:** exact by sorting example wrt $\langle \mathbf{w}; \phi(o_i) \rangle$ [YFRJ07]
- **LAI** with Average Precision (AP) loss: $\Delta_{AP}(y_i, y) = 1 - AP(y)$
 - Δ_{AP} : no linear decomposition wrt examples \neq AUC (ROC)
 - Optimal greedy algorithm in $O(n \log(n))$ [YFRJ07]
 - Speed-up in NIPS'14 [MJK14]

Structured prediction with latent variables

Weakly Supervised Learning (WSL)

- Full annotations expensive \Rightarrow training with weak supervision



- Incorporating latent variables $\mathbf{h} \in \mathcal{H}$

| Variable | Notation | Space | Train | Test |
|----------|--------------|---------------|------------|------------|
| Input | \mathbf{x} | \mathcal{X} | observed | observed |
| Output | \mathbf{y} | \mathcal{Y} | observed | unobserved |
| Latent | \mathbf{h} | \mathcal{H} | unobserved | unobserved |

Structured prediction with latent variables

Latent Structural SVM [YJ09]

- Prediction function : $(\hat{\mathbf{y}}, \hat{\mathbf{h}}) = \arg \max_{(\mathbf{y}, \mathbf{h}) \in \mathcal{Y} \times \mathcal{H}} \langle \mathbf{w}, \Psi(\mathbf{x}, \mathbf{y}, \mathbf{h}) \rangle = \arg \max_{(\mathbf{y}, \mathbf{h}) \in \mathcal{Y} \times \mathcal{H}} s(\mathbf{y}, \mathbf{h})$
 - Joint inference in the $(\mathcal{Y} \times \mathcal{H})$ space
- Training: a set of N labeled trained pairs $(\mathbf{x}_i, \mathbf{y}_i)$
- Training objective: upper bound of $\Delta(\hat{\mathbf{y}}_i, \mathbf{y}_i)$:

$$\frac{1}{2} \|\mathbf{w}\|^2 + \frac{C}{N} \sum_{i=1}^N \max_{(\mathbf{y}, \mathbf{h}) \in \mathcal{Y} \times \mathcal{H}} [\Delta(\mathbf{y}_i, \mathbf{y}) + \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}, \mathbf{h}) \rangle] - \max_{\mathbf{h} \in \mathcal{H}} \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}_i, \mathbf{h}) \rangle$$

- Difference of Convex Functions, solved with CCCP
- LAL: $\max_{(\mathbf{y}, \mathbf{h}) \in \mathcal{Y} \times \mathcal{H}} [\Delta(\mathbf{y}_i, \mathbf{y}) + s(\mathbf{y}, \mathbf{h})]$
 - Challenge exacerbated in the latent case, $(\mathcal{Y} \times \mathcal{H})$ space
 - No exact solution for structured AP ranking [BMJK15]
 \Rightarrow Approximate solution in [BMJK15]

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MANTRA: Minimum Maximum Latent Structural SVM

MANTRA model

- Pair of latent variables $(\mathbf{h}_{i,\mathbf{y}}^+, \mathbf{h}_{i,\mathbf{y}}^-)$
 - **max** scoring latent value: $\mathbf{h}_{i,\mathbf{y}}^+ = \arg \max_{\mathbf{h} \in \mathcal{H}} \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}, \mathbf{h}) \rangle$
 - **min** scoring latent value: $\mathbf{h}_{i,\mathbf{y}}^- = \arg \min_{\mathbf{h} \in \mathcal{H}} \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}, \mathbf{h}) \rangle$

- New scoring function:

$$\begin{aligned} D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}) &= \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}, \mathbf{h}_{i,\mathbf{y}}^+) \rangle + \langle \mathbf{w}, \Psi(\mathbf{x}_i, \mathbf{y}, \mathbf{h}_{i,\mathbf{y}}^-) \rangle \\ &= s(\mathbf{y}, \mathbf{h}_{\mathbf{y}}^+) + s(\mathbf{y}, \mathbf{h}_{\mathbf{y}}^-) \end{aligned} \quad (1)$$

- **MANTRA**: max+min vs max for **LSSVM** \Rightarrow **negative evidence**
- Prediction function \Rightarrow find the output with maximum score

$$\hat{\mathbf{y}} = \arg \max_{\mathbf{y} \in \mathcal{Y}} D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}) \quad (2)$$

MANTRA: Model Training

Learning formulation

- Loss function: $\ell_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}_i) = \max_{\mathbf{y} \in \mathcal{Y}} [\Delta(\mathbf{y}_i, \mathbf{y}) + D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y})] - D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}_i)$
- (Margin rescaling) upper bound of $\Delta(\mathbf{y}_i, \hat{\mathbf{y}})$, constraints:

$$\forall \mathbf{y} \neq \mathbf{y}_i, \quad \underbrace{D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}_i)}_{\text{score for ground truth output}} \geq \underbrace{\Delta(\mathbf{y}_i, \mathbf{y})}_{\text{margin}} + \underbrace{D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y})}_{\text{score for other output}}$$

- Non-convex optimization problem

$$\min_{\mathbf{w}} \frac{1}{2} \|\mathbf{w}\|^2 + \frac{C}{N} \sum_{i=1}^N \ell_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y}_i) \quad (3)$$

- Solver: non convex one slack cutting plane [DA12]
 - Fast convergence
 - Direct optimization \neq CCCP for LSSVM
 - Still needs to solve LAI: $\max_{\mathbf{y}} [\Delta(\mathbf{y}_i, \mathbf{y}) + D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y})]$

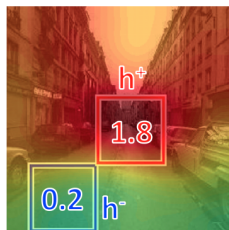
MANTRA: Model & Training Rationale

Intuition of the $\max + \min$ prediction function

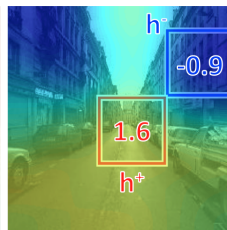
- x image, h image region, y image class
- $\langle w, \Psi(x_i, y, h) \rangle = s(y, h)$: region h score for class y : heatmap
- $s(y) = s(y, h_y^+) + s(y, h_y^-)$
 - h_y^+ : **presence** of class $y \Rightarrow$ large for y_i
 - h_y^- : **localized evidence of the absence** of class y
 - Not too low for $y_i \Rightarrow$ latent space regularization
 - Low for $y \neq y_i \Rightarrow$ tracking negative evidence [PVZF15]



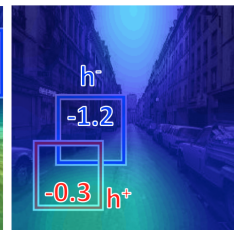
street image x



$D_w(x, \text{street}) = 2$



$D_w(x, \text{highway}) = 0.7$



$D_w(x, \text{coast}) = -1.5$

Intuition in other non-visual contexts, MIL, $h \leftrightarrow$ localization

- Text classification: example with recipe webpages (VISIIR)
 - x recipe text (steps of recipe), h recipe step, y recipe label
 - Lasagna recipe:

LASAGNE MODEL



| Prep | Cook | Ready In |
|------|------|----------|
| 10 m | 50 m | 1 h |



Preheat oven to 375 degrees F (190 degrees C).



Bring a large pot of lightly salted water to a boil. Add pasta and cook for 8 to 10 minutes or until al dente; drain.



In a blender or with an electric mixer, blend mushroom soup, cream of chicken soup and milk until smooth. Cut sausage in half lengthwise and slice thinly.



In a 9x13 inch dish, layer 1 cup soup mixture, 3 noodles, half the sauerkraut, half the sausage and a third of the cheese. Repeat. Top with remaining 3 noodles and remaining soup mixture. Cover with foil.



Bake in preheated oven 25 minutes, then uncover and bake 15 minutes more. Sprinkle with remaining cheese when still hot.

PIZZA MODEL



| Prep | Cook | Ready In |
|------|------|----------|
| 10 m | 50 m | 1 h |



Preheat oven to 375 degrees F (190 degrees C).



Bring a large pot of lightly salted water to a boil. Add pasta and cook for 8 to 10 minutes or until al dente; drain.



In a blender or with an electric mixer, blend mushroom soup, cream of chicken soup and milk until smooth. Cut sausage in half lengthwise and slice thinly.



In a 9x13 inch dish, layer 1 cup soup mixture, 3 noodles, half the sauerkraut, half the sausage and a third of the cheese. Repeat. Top with remaining 3 noodles and remaining soup mixture. Cover with foil.



Bake in preheated oven 25 minutes, then uncover and bake 15 minutes more. Sprinkle with remaining cheese when still hot.

- h_{pizza}^- (boil, water): **negative evidence for class pizza**
- Molecule, e.g. x DNA, h DNA region, y chemical property
 - h^- inhibition region in DNA for the chemical property

MANTRA: Optimization

- MANTRA Instantiation: define $(\mathbf{x}, \mathbf{y}, \mathbf{h})$, $\Psi(\mathbf{x}, \mathbf{y}, \mathbf{h})$, $\Delta(\mathbf{y}_i, \mathbf{y})$
- Instantiations: binary & multi-class classification, AP ranking

| | Binary | Multi-class | AP Ranking |
|--|---|---|-------------------------------------|
| \mathbf{x} | bag (image/text/molecule) | bag (set of regions) | set of bags (of regions) |
| \mathbf{y} | ± 1 | $\{1, \dots, K\}$ | ranking matrix |
| \mathbf{h} | instance (region) | region | regions |
| $\Psi(\mathbf{x}, \mathbf{y}, \mathbf{h})$ | $\mathbf{y} \cdot \phi(\mathbf{x}, \mathbf{h})$ | $\{I(\mathbf{y} = 1)\Phi(\mathbf{x}, \mathbf{h}), \dots, I(\mathbf{y} = K)\Phi(\mathbf{x}, \mathbf{h})\}$ | joint latent ranking feature map |
| $\Delta(\mathbf{y}_i, \mathbf{y})$ | 0/1 loss | 0/1 loss | AP loss |
| LAI | exhaustive | exhaustive | exact and efficient |

- Solve Inference $\max_{\mathbf{y}} D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y})$ & LAI $\max_{\mathbf{y}} [\Delta(\mathbf{y}_i, \mathbf{y}) + D_{\mathbf{w}}(\mathbf{x}_i, \mathbf{y})]$
 - Exhaustive for binary/multi-class classification
 - **Exact** and **efficient solutions** for ranking

MANTRA: Optimization

Latent structured AP ranking

- Latent feature map: $\Psi(\mathbf{x}, \mathbf{y}, \mathbf{h}) = \sum_{x_i \in \oplus} \sum_{x_j \in \ominus} y_{ij} [\Phi(x_i, h_{i,j}) - \Phi(x_j, h_{j,i})]$
 - $D(\mathbf{x}_i, \mathbf{y}) = \max_h \langle \mathbf{w}; \Psi(\mathbf{x}, \mathbf{y}, \mathbf{h}) \rangle + \min_h \langle \mathbf{w}; \Psi(\mathbf{x}, \mathbf{y}, \mathbf{h}) \rangle$
- **Lemma:** $D(\mathbf{x}_i, \mathbf{y}) = \sum_{x_i \in \oplus} \sum_{x_j \in \ominus} y_{ij} [\langle \mathbf{w}, \Phi_-^+(x_i) \rangle - \langle \mathbf{w}, \Phi_-^+(x_j) \rangle]$
 - $\langle \mathbf{w}, \Phi_-^+(x_i) \rangle = \max_{h \in \mathcal{H}_i} \langle \mathbf{w}, \Phi(x_i, h) \rangle + \min_{h \in \mathcal{H}_i} \langle \mathbf{w}, \Phi(x_i, h) \rangle$
 - ~ Supervised problem with feature for each example \mathbf{x}_i : $\Phi_-^+(x_i)$
- ▷ **Elegant symmetrization due to the max+min scoring**
- ▷ **Decoupling optimization over \mathbf{y} and \mathbf{h} , \neq [YJ09, BMJK15]**
- Inference: sort examples wrt $\langle \mathbf{w}, \Phi_-^+(x_i) \rangle$ scores
- LAI: ~ supervised problem with $\Phi_-^+(x_i)$ feature for each \mathbf{x}_i , use [YFRJ07]

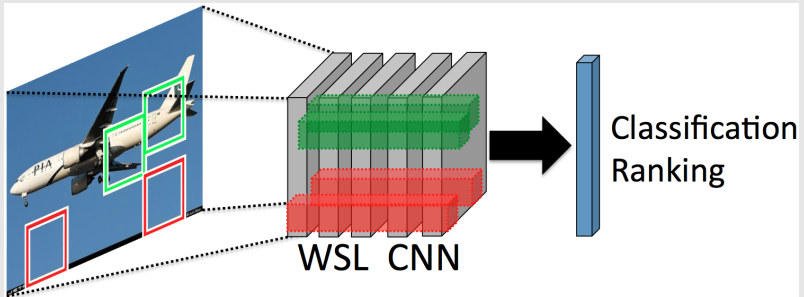
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WELDON

Weakly Supervised Learning of Deep Convolutional Neural Networks

- MANTRA extension for training deep CNNs



- learning $\Psi(\mathbf{x}, \mathbf{y})$: end-to-end WSL of deep CNNs with structured prediction
 - Incorporating multiple positive & negative evidence
 - Training deep CNNs with structured loss
 - Architectural choices \Rightarrow efficiency & robustness to over-fitting

WELDON: Model & Training

Region selection policy: k-max + k-min pooling

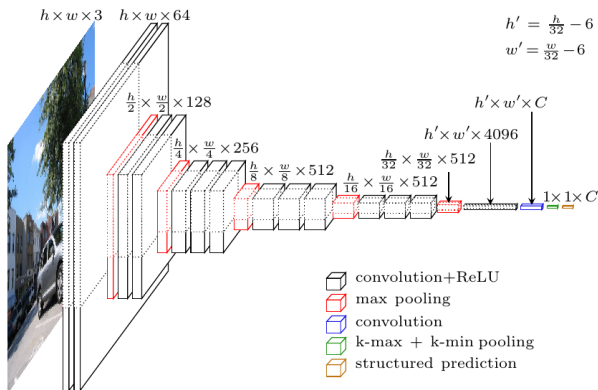
- Top-instances selection [LV15]: \sum k-max scores \Rightarrow convex
- Adding k-min (negative evidence): \sum k-min scores \Rightarrow concave
- Using more instances \Rightarrow robustness to outliers



Training: optimization for structured ranking

- MANTRA generalization for k-max + k-min: exact solutions
 - Inference: sorting wrt k-max + k-min scores
 - LAI: each example represented by k-max + k-min features

WELDON: WSL deep architecture



- Convolutional architecture
 - Efficient region feature computation
 - ImageNet transfer
- Fine-tuning
 - ⇒ end-to-end training
- MATRA + top instances
 - ⇒ k-max + k-min
- Structured ranking AP loss for k-max + k-min

WELDON Weakly Supervised Learning Insight

class is present: **Increase** score of selecting windows



class is absent: **Decrease** score of selecting windows



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Negative Evidence Models: Results

Multiple Instance Learning (MIL)

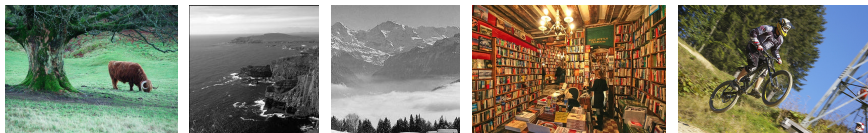
- MIL datasets, binary classification: image, text & molecule
- $\Psi(x, y, h)$: handcrafted features describing instances in bags
 - Image region descriptor, BoW for text passage *etc*

| Method | Image | Musk | Text |
|--------------|-------------|-------------|-------------|
| mi-SVM | 73.4 | 84.5 | 81.6 |
| MI-SVM | 75.5 | 81.7 | 80.3 |
| LSVM | 74.4 | 82.7 | 80 |
| SyMIL | 80.2 | 89.2 | 84.8 |
| MICA | 73.9 | 87.5 | 82.3 |
| MIGraph | 76.1 | 90 | - |
| MI-CRF | 78.5 | 86.7 | - |
| GP-WDA | 79 | 88.4 | 83.2 |
| eMIL | 77 | 85.3 | 82.7 |



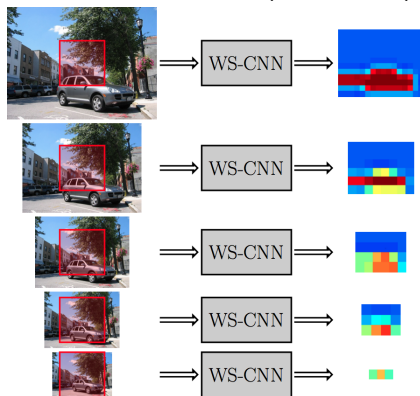
- $\max + \min \gg \max$
- \sim state-of-the-art results with more complex models (MI-CRF, MIGraph)

Negative Evidence Models: Visual Recognition Results



- $\Psi(x, y, h)$: deep features on regions
MANTRA transfer (ImageNet, Places)
WELDON fine-tuning (target dataset)
- Instantiations: Multi-class classification
& ranking

- Multi-scale: 8 scales (Object Bank)



| Dataset | # ex | # class | Eval |
|-----------|------|---------|------|
| VOC07 | 10k | 20 | AP |
| VOC12 | 10k | 20 | AP |
| 15 Scene | 5k | 15 | MC |
| MIT67 | 7k | 67 | MC |
| VOC12 act | 4k | 10 | AP |
| COCO | 120k | 80 | AP |

Negative Evidence Models: Visual Recognition Results

State-of-the-art results

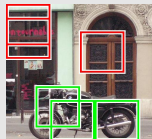
| Multi-label (mAP) | VOC 2007 | VOC 2012 |
|-------------------|--------------|-------------|
| VGG16 | 84.5 | 82.8 |
| SPP net | 82.4 | |
| Deep WSL MIL | | 81.8 |
| MANTRA | 85.8 | |
| WELDON | 90.2 | 88.5 |
| Multi-label (mAP) | VOC12 Action | COCO |
| VGG16 | 67.1 | 59.7 |
| Deep WSL MIL | | 62.8 |
| WELDON | 75.0 | 68.8 |
| Multi-class (acc) | 15 Scene | MIT67 |
| VGG16 | 91.2 | 69.9 |
| MOP CNN | | 68.9 |
| MANTRA | 93.3 | 76.6 |
| Negative parts | | 77.1 |
| WELDON | 94.3 | 78.0 |

Negative Evidence Models: Results

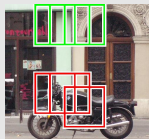
Impact of the different improvements

| a) max | b) +k=3 | c) +min | d) +AP | VOC07 | VOC12 action |
|--------|---------|---------|--------|-------------|--------------|
| ✓ | | | | 83.6 | 53.5 |
| ✓ | ✓ | | | 86.3 | 62.6 |
| ✓ | | ✓ | | 87.5 | 68.4 |
| ✓ | | ✓ | ✓ | 88.4 | 71.7 |
| ✓ | ✓ | ✓ | | 87.8 | 69.8 |
| ✓ | ✓ | ✓ | ✓ | 88.9 | 72.6 |

Detection results ??



Motorbike (1.1)



Sofa (-0.8)

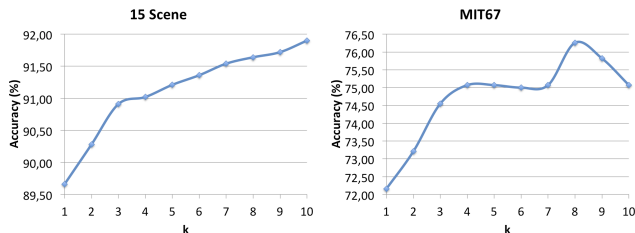


Sofa (1.2)



Horse (-0.6)

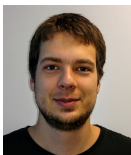
Negative Evidence Models: Visual Results



Take-home message: Contributions at different levels:

- Model: prediction function $\max + (k - \min) > \max$
 - Using (k)-top-instances help, but selection needed
- Weakly supervised learning
 - AP ranking optimization: AP loss > Acc loss
- Deep CNN extension: learning $\Psi(\mathbf{x}, \mathbf{y})$

Future Works: Exploring other structured output predictions tasks, e.g. semantic segmentation



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MANTRA project page
<http://webia.lip6.fr/~durandt/project/mantra.html>



Thibaut Durand, Nicolas Thome, and Matthieu Cord.

WELDON: Weakly Supervised Learning of Deep Convolutional Neural Networks.
In IEEE Computer Vision and Pattern Recognition (CVPR), 2016.



Thibaut Durand, Nicolas Thome, and Matthieu Cord.

MANTRA: Minimum Maximum LSSVM for Image Classification and Ranking.
In IEEE International Conference on Computer Vision (ICCV), 2015.

References I



Aseem Behl, Prithish Mohapatra, C. V. Jawahar, and M. Pawan Kumar, *Optimizing average precision using weakly supervised data*, IEEE Trans. Pattern Anal. Mach. Intell. **37** (2015), no. 12, 2545–2557.



Liang-Chieh Chen, Alexander G. Schwing, Alan L. Yuille, and Raquel Urtasun, *Learning deep structured models*, Proceedings of the 32nd International Conference on Machine Learning, ICML 2015, Lille, France, 6-11 July 2015, 2015, pp. 1785–1794.



Trinh-Minh-Tri Do and Thierry Artières, *Regularized bundle methods for convex and non-convex risks*, JMLR (2012).



Weixin Li and Nuno Vasconcelos, *Multiple instance learning for soft bags via top instances*, The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2015.



Prithish Mohapatra, C.V. Jawahar, and M. Pawan Kumar, *Efficient optimization for average precision svm*, NIPS, 2014.



S. N. Parizi, A. Vedaldi, A. Zisserman, and P. F. Felzenszwalb, *Automatic discovery and optimization of parts for image classification*, Proceedings of the International Conference on Learning Representations (ICLR), 2015.



Ioannis Tsochantaridis, Thorsten Joachims, Thomas Hofmann, and Yasemin Altun, *Large margin methods for structured and interdependent output variables*, Journal of Machine Learning Research, 2005, pp. 1453–1484.

References II



Yisong Yue, Thomas Finley, Filip Radlinski, and Thorsten Joachims, *A support vector method for optimizing average precision*, Proceedings of the 30th annual international ACM SIGIR conference on Research and development in information retrieval, ACM, 2007, pp. 271–278.



Chun-Nam Yu and T. Joachims, *Learning structural svms with latent variables*, ICML, 2009.